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On planar fermions with quartic interaction at finite temperature and density

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Abstract. We study the breaking of parity symmetry in the 2+1 Gross-Neveu model at finite temperature with chemical potential μ, in the presence of an external magnetic field. We find that the requirement of gauge invariance, which is considered mandatory in the presence of gauge fields, breaks parity at any finite temperature and provides for dynamical mass generation, preventing symmetry restoration for any non-vanishing μ. The dynamical mass becomes negligibly small as temperature is raised.

PACS. 11.10.Kk Field theories in dimensions other than four – 11.30.Er Charge conjugation, parity, time reversal, and other discrete symmetries – 71.27.+a Strongly correlated electron systems; heavy fermions

1 Introduction

The study of planar fermion systems has become an active area of research in the last years, on its own right and because of the many applications in strongly correlated electron systems as high T_c superconductors [1], quantum Hall effect systems [2], layered graphite [3], etc. A frequent situation includes quartic fermion interaction which can be modeled by a Gross-Neveu type Lagrangian with fermions of zero bare mass. One of the relevant issues within this context is the opening of a gap for the quasiparticles described by this effective model due to self-interactions, the presence of an external magnetic field or finite temperature and density effects.

Our main point in the present paper is that in the presence of an external magnetic field, fermionic fields should be quantized in a gauge-invariant framework and hence, for an irreducible representation of two component fermions, a parity anomaly will naturally appear [4]. The parity breaking effect of course depends on the external field configuration and is trivial (unobservable) in the case of constant magnetic fields, unless a non vanishing chemical potential (μ) is considered. Previous analysis [5–8] have either considered the case with $\mu = 0$ or have duplicated the number of fermion components in such a way that the parity anomaly cancels out. Our analysis is applicable to generic planar fermion systems.

We show that, at finite quasi-particle density, the term that breaks parity in the effective action changes dramatically the analysis of the gap equation. In particular, the opening of a gap as a function of the external magnetic field occurs at any finite temperature, thus providing for dynamical mass generation and preventing symmetry restoration for any non-vanishing μ. It should be pointed that the effect of this correction is of order $1/T$ and the gapless behaviour is recovered at high temperatures.

2 The model

Let us consider the following (Euclidean) 3-dimensional fermionic Lagrangian with four fermion interaction

$$\mathcal{L} = \bar{\psi}^a(\partial\!\!\!/ + \mathrm{i}e\, A\!\!\!/ + \gamma^0\mu)\psi^a + \frac{g_0}{2N}(\bar{\psi}^a\psi^a)^2, \qquad (1)$$

where $a = 1, \cdots, N$ is a flavor index and the Fermi fields are in an irreducible two component spinor representation. This model field theory is known as the Gross-Neveu model. The gauge field A_ν ($\nu = 0, 1, 2$) represents an external background that for a constant transverse magnetic field B can be chosen as *e.g.* $A_0 = 0$, $A_i = -Bx_2\delta_{i1}$.

Apart from the usual gauge invariance $A_\nu \to A_\nu^{(\lambda)} = A_\nu + \mathrm{e}^{-1}\partial_\nu\lambda$, $\psi \to \psi^{(\lambda)} = \exp(-\mathrm{i}\lambda)\psi$, the theory defined

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by equation (1) is at the classical level invariant under parity transformations, which are defined as

$$\begin{aligned}(x_0, x_1, x_2) &\to (x_0, -x_1, x_2),\\ (A_0, A_1, A_2) &\to (A_0, -A_1, A_2)\\ \psi &\to \gamma_1\psi.\end{aligned} \tag{2}$$

Now, since in odd space-time dimensions the path integration measure cannot be defined in a way that preserves both gauge and parity invariance, a parity violating contribution can arise if one adopts a gauge invariant quantization. Indeed, from the definition of the partition function $\mathcal{Z}[A_\nu]$, if $\mathcal{Z}[A_\nu] = \mathcal{Z}[A_\nu^{(\lambda)}]$ one necessarily has to impose invariance of the fermionic measure under gauge transformations $\psi \to \exp(-\mathrm{i}\lambda)\psi$. Due to the presence of an external magnetic field we consider mandatory to quantize the theory in a gauge invariant way, which then leads to the well known parity anomaly [4].

It is also known that the parity anomaly can be overcome by a slight change in the theory, consisting in the use of a suitable reducible four component spinor representation for the Fermi fields (as done in [9]). It should be stressed that this change not merely duplicates the number of components, but does also change the interaction term [6]. We analyze in the following the case in which an irreducible spinor representation and a gauge invariant regularization are chosen.

In order to study the opening of a gap in this system, which is associated with the breaking of parity symmetry, we use the $1/N$ standard procedure to compute the effective potential for the fermion system. One first introduces an auxiliary field σ trading the quartic interaction term for a linear σ vertex

$$\mathcal{L} = \bar{\psi}^a(\partial\!\!\!/ + \mathrm{i}e\, A\!\!\!/ + \gamma^0\mu + \sigma)\psi^a - \frac{N}{2g_0}\sigma^2; \tag{3}$$

the equation of motion for σ sets the constraint

$$\sigma = \frac{g_0}{N}\bar{\psi}^a\psi^a. \tag{4}$$

Parity invariance of (3) at the classical level (or alternatively consistency of Eq. (4)) requires that the field σ changes as a pseudo-scalar under parity,

$$\sigma \to -\sigma. \tag{5}$$

The breaking of parity symmetry at the quantum level would be now signaled by a non vanishing expectation value of the fermion condensate. Though the scalar field σ should be integrated out at some stage of the computation, it is well known that the fermion condensate expectation value can be computed to leading order in $1/N$ by considering constant values for σ [10].

The effective potential is defined as

$$V^{\mathrm{eff}}_{\beta,\mu}[\sigma] \equiv -\frac{1}{\beta L^2}\log\left(\int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp - \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^2x \times\left(\bar{\psi}^a\left(\partial\!\!\!/ + \mathrm{i}e\, A\!\!\!/ + \gamma^0\mu + \sigma\right)\psi^a - \frac{N}{2g_0}\,\sigma^2\right)\right) \tag{6}$$

and the vacuum expectation for the fermion condensate can be found from its minima, that is solving the gap equation $\delta V^{\mathrm{eff}}/\delta\sigma = 0$.

We distinguish two different contributions to the effective potential, one even in σ defined as

$$V^{\mathrm{even}}[\sigma] \equiv \frac{1}{2}\left(V^{\mathrm{eff}}[\sigma] + V^{\mathrm{eff}}[-\sigma]\right) \tag{7}$$

and the other odd in σ, which signals the breaking of parity, defined as

$$V^{\mathrm{odd}}[\sigma] \equiv \frac{1}{2}(V^{\mathrm{eff}}[\sigma] - V^{\mathrm{eff}}[-\sigma]). \tag{8}$$

The first contribution, V^{even}, can be computed by any method that assumes that V^{eff} depends on σ^2; in particular, a detailed computation was performed in [9] using the Schwinger proper time method. The regularized result is

$$\begin{aligned}V^{\mathrm{even}}_{\beta,\mu}[\sigma] = &\\ \frac{N}{2\pi}&\left[\frac{\Lambda}{2\sqrt{\pi}}(\frac{2\sqrt{\pi}}{g} - 1)\sigma^2 - \frac{\sqrt{2}}{l^3}\zeta(-\frac{1}{2}, \frac{(\sigma l)^2}{2} + 1) - \frac{|\sigma|}{2l^2}\right]\\ -\frac{N}{4\pi\beta l^2}&\left\{\log\left(1 + \exp(-2\beta|\sigma|) + 2\exp(-\beta|\sigma|)\cosh(\beta\mu)\right)\right.\\ &+ 2\sum_{n=1}^{\infty}\log\left(1 + \exp\left(-2\beta\sqrt{\sigma^2 + \frac{2n}{l^2}}\right)\right.\\ &\left.\left.+2\exp\left(-\beta\sqrt{\sigma^2 + \frac{2n}{l^2}}\right)\cosh(\beta\mu)\right)\right\},\end{aligned} \tag{9}$$

where $l = 1/\sqrt{|eB|}$, $g = N\Lambda g_0/\pi$ and Λ is an UV cutoff.

The parity violating contribution to the effective action for a fermion system at finite temperature in a gauge background has been recently computed in exact form for constant field strength configurations in [11]. The complete expression of the parity breaking term which is generated after fermionic integration has a closed form which reduces to the previously obtained expression [12] in a perturbative expansion in powers of the gauge field [11]. In the case of zero temperature one recovers the topological invariant Chern-Simons action. The above mentioned result, originally presented without consideration of a chemical potential, can be straightforwardly applied to the case at hand, since the chemical potential μ in equation (6) plays the same role as an imaginary time component of the gauge field and a constant σ plays the role of a mass term. In fact, replacing the time component eA_0 in [11] by $i\mu$ one gets

$$V^{\mathrm{odd}}_{\beta,\mu}[\sigma] = -\frac{N}{2\pi\beta l^2}\ \mathrm{arctanh}\left(\tanh\left(\frac{\beta\sigma}{2}\right)\tanh\left(\frac{\beta\mu}{2}\right)\right). \tag{10}$$

The complete expression for the effective potential is of course the sum of both contributions, $V^{\mathrm{eff}}_{\beta,\mu}[\sigma] = V^{\mathrm{even}}_{\beta,\mu}[\sigma] + V^{\mathrm{odd}}_{\beta,\mu}[\sigma]$.

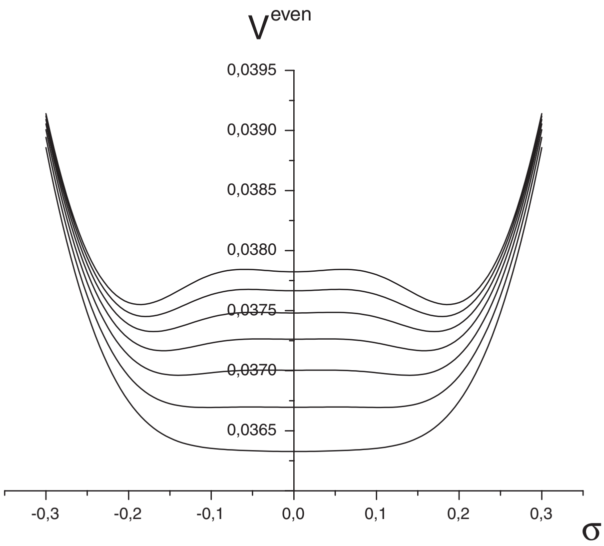

Fig. 1. Parity conserving effective potential, showing the transition from massless to massive regime. Coupling g, magnetic field and chemical potential are kept fixed. Higher temperatures show symmetry restoration (lower curves). All quantities are measured in an arbitrary mass scale m_0: $V^{\text{even}} \to \pi V^{\text{even}}/m_0^3$, $\sigma \to \sigma/m_0$.

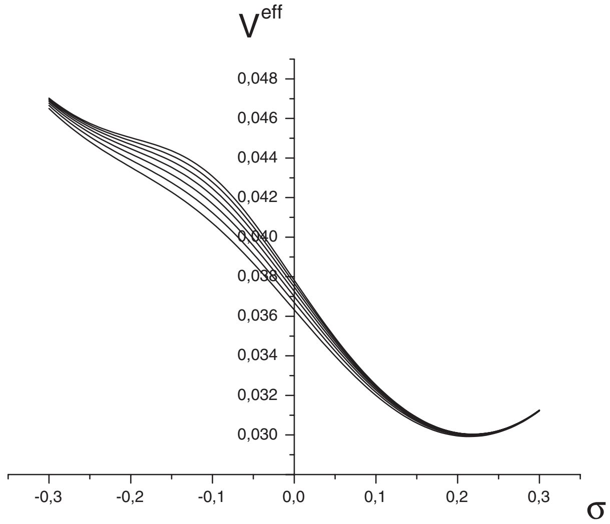

Fig. 2. Complete effective potential, showing persistence of symmetry breaking for the same range parameters of Figure 1. Temperature grows from top to bottom. All quantities are measured in a mass scale m_0: $V^{\text{eff}} \to \pi V^{\text{even}}/m_0^3$, $\sigma \to \sigma/m_0$.

It is now easy to see that the term in (10) changes the mass generation picture completely at low temperatures. This is because it is smooth at the origin and odd in σ and hence shifts the minimum of the effective potential away from zero, leading to a mass gap. A numerical analysis of the gap equation $\delta V^{\text{eff}}/\delta\sigma = 0$ confirms that there is no minimum at $\sigma = 0$, except at very high temperatures, where V^{odd} is subdominant (of order $1/T$) with respect to the even terms.

In order to explore the meaning of V_{eff} we show in the following plots striking qualitative differences between the gauge invariant and the parity conserving effective actions. In Figure 1 we show the parity conserving effective potential for fixed magnetic field and chemical potential for a range of temperatures where the transition between massless and massive regimes is apparent. In Figure 2 we plot the gauge invariant effective potential for the same range of parameters, in which case the theory is always massive. In Figure 3 we include higher temperatures so as to show the tendency to symmetry restoration.

In all figures we plot dimensionless quantities in terms of an arbitrary mass scale. As an example we choose $\Lambda = \sqrt{\pi}$, $g = 0.9\sqrt{\pi}$, $\mu = 0.1$ and $eB = 1$ [9].

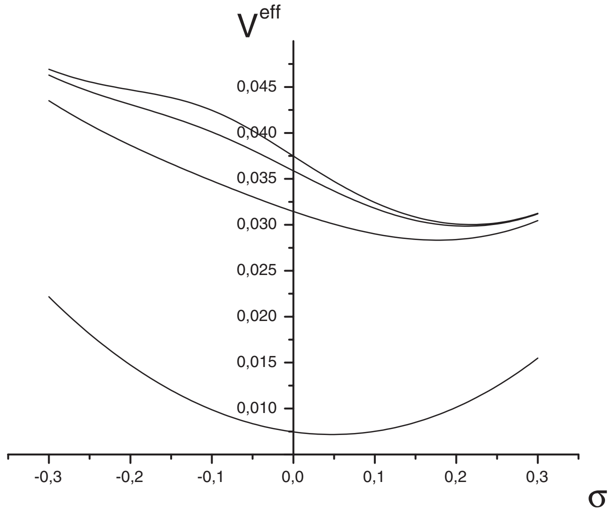

Fig. 3. Complete effective potential for a wider range of temperatures, indicating the irrelevance of the parity breaking contribution at high temperature (lower curve). All quantities are measured in a mass scale m_0: $V^{\text{eff}} \to \pi V^{\text{even}}/m_0^3$, $\sigma \to \sigma/m_0$.

3 Conclusions

In the present paper we have analyzed the consequences of the parity anomaly on the usual picture of dynamical mass generation (associated with parity symmetry breaking) for planar fermions in an irreducible two component fermion representation with quartic interactions at finite temperature and density. Our main observation is that the inclusion of a finite chemical potential prevents the appearance of a symmetric phase for arbitrarily small magnetic fields.

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